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MODELING OF FLOW IN A NEAR-WELLBORE NETWORK

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ABSTRACT

We consider steady-state multiphase flow in the near-well region of a completed horizontal well. The flow topography in this system is such that many alternate paths are available for fluid to travel from the reservoir to the producing vertical wellbore. Predicting and controlling this flow is essential to optimizing recovery from the reservoir.

We treat the system as a pipe network. We decouple the mass conservation and pressure equations and solve for the phase splits at each junction in the network under the assumption that there is complete mixing at each branch point. Thus, the gas-liquid ratio (GLR) and water oil ratio (WOR) of each stream exiting a given network junction is constant and is determined by the quality of the streams entering the junction. (This assumption is reasonable since the flow paths in the "network" are short.) We use Newton iteration to solve the pressure equations. The resulting algorithm is fast and robust, so that it is well suited for coupling with a reservoir flow simulator. We illustrate the method by presenting an example.

INTRODUCTION

Many modern oil wells are drilled with long horizontal (or near-horizontal) laterals. This design maximizes exposure of the reservoir to the wellbore by providing a large area of contact between the reservoir rock and the wellbore, which in turn facilitates fluid transfer from the reservoir to the well. In long, small-diameter well laterals, a large pressure drop may be generated by axial flow of fluids from the "toe" to the "heel" of the well, so that reservoir rock in the vicinity of the well "heel" may be exposed to a large (radial) pressure drop driving fluid toward the wellbore, whereas reservoir areas in the vicinity of the "toe" of the well may only experience small pressure drops. The result is that a large amount of reservoir fluid may enter the well "heel", while very little enters at the "toe". This is problematic for several reasons; first, the reservoir is only effectively producing from the heel of the well, with very little drainage from the toe. This implies that the effective well length is much shorter than the actual well length. Second, there is a great likelihood of water or gas "coning" near the heel of the well. This is undesirable, because premature removal of gas from the reservoir severely depletes the reservoir pressure, and thus its ability to flow fluids to the surface. In addition, production capacity may be limited by gas processing capacity. Also, excessive water production is undesirable since it is costly to lift to the surface where it must be treated before being disposed of.

One approach to lessening the impact of uneven inflow to the well has been to employ ICD's (Inflow Control Devices) [1]; these are essentially pressure loss devices that are positioned between the producing wellbore and the reservoir in an effort to insulate the reservoir from the wellbore pressures. Under ideal conditions, ICD's cause the reservoir to experience no pressure variation along the well length, allowing flow from the reservoir to the wellbore to be evenly distributed, and avoiding coning of oil and water at the heel. With the introduction of these and other inflow control devices, modern oil wells are no longer simple perforated pipes that allow entry of reservoir fluids along their length. Instead, wells with complex completion strategies offer a variety of possible fluid paths for flow of the reservoir fluids from the reservoir through the "near-well network" of annuli, ICD's, chokes and base pipe to the vertical well tubing. This work addresses the modeling of the flow of oil, water and gas through this network to the heel of the well. Our objective is to obtain an efficient, robust model that could be easily coupled with a reservoir simulator to design and optimize well completion strategies for long horizontal wells.

NOMENCLATURE

GLR	Ratio of gas rate to liquid rate,
$\dot{m}$	Mass rate, lbm/day
p	Pressure, psia
q	Volumetric rate, scf/day
WOR	Ratio of water rate to oil rate.

Greek

Δ	Change
ρ	Density, lbm/ft^3

Subscripts

i	"node" or "arc" index
g	Gas
o	Oil
sc	Standard conditions
w	Water

MODEL ASSUMPTIONS AND APPROACH

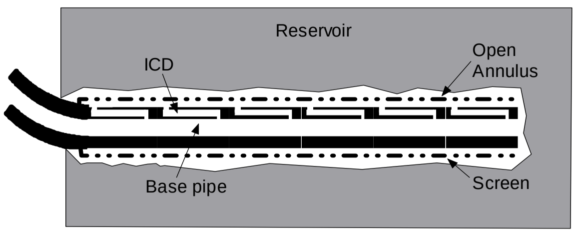

Fig. 1 – Schematic of a Completed Well in a Petroleum Reservoir

Figure 1 shows a schematic of flow to a horizontal well through an open-hole completion, sandscreen, and annulus, through Inflow Control Devices (ICD's) to the basepipe. Clearly, there are a multitude of paths available for a fluid particle to follow on its way from the reservoir to the heel of the well; further, different phases are free to split at each flow junction and follow different paths.

We assume isothermal flow, and that at any instant, the horizontal-well "network" is at steady state; i.e., there is no storage of fluids in this region. (This assumption is reasonable since transient times in the well network are small compared to the oil reservoir.) For ease of illustration, we assume that there is no axial flow in the open annulus on the exterior of the screen, and we represent the near-wellbore well-reservoir system by the network shown in Fig. 2. (If axial flow in the open annulus were considered, we would simply add in an extra annulus layer in Fig. 2)

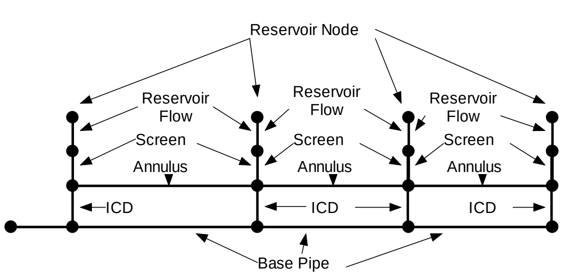

Fig. 2 – Well Network Schematic.

In Fig. 2, the black circles represent "nodes" or points at which flow paths originate or connect to different flow paths, and the lines are "arcs" or flow paths between nodes, (see Refs. [2] - [3]); pressure differences between the ends of a flow path occur whenever mass is flowing in the arc. For steady-state flow of an incompressible fluid through a network of pipes, Acton [4] showed that the solution of mass rate and pressure distribution in the network could be found by solving mass balance and pressure balance equations in a fraction of the total number of nodes and/or arcs. Our solution for multiphase flow in networks builds on Acton's solution. For the purpose of illustration, we demonstrate Acton's approach to solving for steady-state flow of an incompressible fluid in the network shown in Fig. 2, then, we extend his ideas to solving for flow in a general multiphase network.

STEADY-STATE INCOMPRESSIBLE FLOW – ACTON'S METHOD

Considering the network in Fig. 2, our first step is to characterize the arcs as either "major" or "minor" arcs; in Fig. 3, the nodes are numbered with letters from "a" – "q", with network boundary nodes at "a" and "n" – "q". Reservoir nodes, i.e., network boundary nodes that are connected to the reservoir are denoted "n" – "q". Arcs are numbered from 0 - 18; major arcs are denoted by solid black lines, and minor arcs are dashed lines. The designation of an arc as "major" or "minor" is somewhat arbitrary as long as the following guidelines are obeyed:

1) All arcs that originate (or terminate) at a reservoir node are minor arcs;
2) It must be possible to get from any non-reservoir node to the outlet node, "a", via a continuous sequence of major arcs.
3) Any arcs that are not necessary to meet the above guidelines are designated as minor arcs.

In the case of Fig. 3, we have 7 minor arcs and 12 major arcs. (Note that the process of selecting major and minor arcs is not unique, however, the results that we obtain in the following are general.)

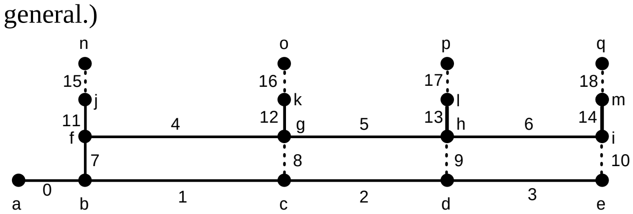

Fig. 3 – Major and Minor Arcs.

We assume that flow is from left to right and from top to bottom; if we are wrong, our calculations will result in negative answers for the mass flow rates. If we consider mass conservation at each interior node, i.e., all nodes excluding the boundary nodes, we obtain the following equations for nodes "b" – "m", respectively:

$$\dot{m}_0 - \dot{m}_1 - \dot{m}_7 = 0 \qquad (1)$$

$$\dot{m}_1 - \dot{m}_2 = \dot{m}_8 \qquad (2)$$

$$\dot{m}_2 - \dot{m}_3 = \dot{m}_9 \qquad (3)$$

$$\dot{m}_3 = \dot{m}_{10} \quad (4)$$

$$\dot{m}_7 - \dot{m}_4 - \dot{m}_{11} = 0 \quad (5)$$

$$\dot{m}_4 - \dot{m}_5 - \dot{m}_{12} = -\dot{m}_8 \quad (6)$$

$$\dot{m}_5 - \dot{m}_6 - \dot{m}_{13} = -\dot{m}_9 \quad (7)$$

$$\dot{m}_6 - \dot{m}_{14} = -\dot{m}_{10} \quad (8)$$

$$\dot{m}_{11} = \dot{m}_{15} \quad (9)$$

$$\dot{m}_{12} = \dot{m}_{16} \quad (10)$$

$$\dot{m}_{13} = \dot{m}_{17} \quad (11)$$

$$\dot{m}_{14} = \dot{m}_{18} \quad (12)$$

In Eqs. 1 - 12, rates in the major arcs are on the left sides and rates in the minor arcs are on the right side. Note that there are 12 equations (one for each major arc), and if the minor arc rates are specified, the major arc rates are uniquely determined. In fact, if Eqs. 1 - 12 are written as a matrix equation, it is only necessary to invert the coefficient matrix for the major arc rates once to obtain the major arc rates from the minor arc rates. This result is general; any steady state network of pipes can be split into major arcs and minor arcs, and a unique relationship exists between rates in the major arcs and those in the minor arcs. Determination of that relationship requires a single matrix inversion.

In order to solve for the minor arc rates, we need to consider pressure loss equations in the system. The following “circuit” equations close the system:

$$\Delta p(\dot{m}_4) + \Delta p(\dot{m}_7) - \Delta p(\dot{m}_1) = \Delta p(\dot{m}_8) \quad (13)$$

$$\Delta p(\dot{m}_5) + \Delta p(\dot{m}_4) + \Delta p(\dot{m}_7) - \Delta p(\dot{m}_1) - \Delta p(\dot{m}_2) = \Delta p(\dot{m}_9) \quad (14)$$

$$\Delta p(\dot{m}_6) + \Delta p(\dot{m}_5) + \Delta p(\dot{m}_4) + \Delta p(\dot{m}_7) - \Delta p(\dot{m}_1) - \Delta p(\dot{m}_2) - \Delta p(\dot{m}_3) = \Delta p(\dot{m}_{10}) \quad (15)$$

$$\Delta p(\dot{m}_{11}) + \Delta p(\dot{m}_7) + \Delta p(\dot{m}_0) = -\Delta p(\dot{m}_{15}) - (p_n - p_a) \quad (16)$$

$$\Delta p(\dot{m}_{12}) + \Delta p(\dot{m}_4) + \Delta p(\dot{m}_7) + \Delta p(\dot{m}_0) = -\Delta p(\dot{m}_{16}) - (p_o - p_a) \quad (17)$$

$$\Delta p(\dot{m}_{13}) + \Delta p(\dot{m}_5) + \Delta p(\dot{m}_4) + \Delta p(\dot{m}_7) + \Delta p(\dot{m}_0) = -\Delta p(\dot{m}_{17}) - (p_p - p_a) \quad (18)$$

$$\Delta p(\dot{m}_{14}) + \Delta p(\dot{m}_6) + \Delta p(\dot{m}_5) + \Delta p(\dot{m}_4) + \Delta p(\dot{m}_7) + \Delta p(\dot{m}_0) = -\Delta p(\dot{m}_{17}) - (p_q - p_a) \quad (19)$$

Equations 13 - 19 are 7 nonlinear equations in the minor arc rates. If the pressures are specified at the outlet (Node “a”) and at the reservoir nodes (“n” – “q”), the unknowns are

$$\{\dot{m}_8, \dot{m}_9, \dot{m}_{10}, \dot{m}_{15}, \dot{m}_{16}, \dot{m}_{17}, \dot{m}_{18}\}.$$

In this case, the algorithm for solving the system is as follows:

- Guess values for the minor arc flow rates;
- Solve Eqs. 1 - 12 for the major arc rates
- Solve Eqs. 13- 19 for the minor arc rates using Newton iteration; at each iteration where minor arc rates are updated, the major arc rates are updated using Eqs. 1 - 12.

If rate is specified at the outlet node, we have an additional rate constraint, and we lose 1 circuit equation. In this case, we convert one of the minor arcs connected to a reservoir node (e.g., arc #15) to a major arc and decrease the number of unknown minor arc rates by 1.

MULTIPHASE FLOW NETWORKS

If the fluid flowing through the network is compressible (e.g., gas) or if multiple phases flow simultaneously through the network, the problem becomes more complicated, since pressure drops in the circuit equations depend on the PVT properties of the fluids flowing in each arc, and these depend on the composition of the fluid flowing and the average pressure in the arc. Regardless of the type of flow (i.e., single or multiphase), the mass balance equations (Eqs. 1-12) are valid; for multiphase flow, these mass balance equations may be applied to total mass flow rate or the mass rate of each flowing component. In the case of multiphase flow, there are additional unknowns: in particular, the manner in which the phases are distributed in the minor arcs. Thus, if we have three components flowing (e.g., oil, water and gas), in addition to the total mass flow rates in each minor arc (from which the major arc rates can be easily calculated), we also have the mass flow rates of two of the components (e.g., oil and water) in each minor arc. This requires that we provide additional constraints: i.e., two for each minor arc for the flow of three components. Unfortunately, at present, there is no unique way to do this. It is well known [5] – [9] that multiphase streams split unevenly when they encounter branches in networks. We assume that in our system, flow distances are so short that the phases mix completely to form a homogeneous mixture that splits evenly at each network branch. We employ the following algorithm:

- At each reservoir node, we specify an inlet Gas-Liquid ratio (GLR) and Water-Oil ratio (WOR); this uniquely quantifies the mass rates of each component in each of the minor arcs connected to the reservoir nodes.
- Starting at each reservoir node, we follow the flow through the network to the outlet, (using the last assumed or computed mass flow directions), and assign GLR’s and WOR’s to each arc along the path using the following rules:
    - If the flow stream encounters a node where flow is diverging, the GLR’s and WOR’s leaving the node are assumed to be equal in each branch (i.e., perfect mixing of the phases at the node) and are determined by the total rates of each component entering the node. An attempt is made to follow the flow from each divergent path to the outlet.
    - If the flow encounters a node where flow is converging and component mass rates are not known from all entering paths, the calculations are abandoned for that flow path.
    - Providing that there is no calculated recirculating flow anywhere in the network (a physical impossibility since there must be a pressure difference to drive the

flow), this procedure results in a unique distribution of components and phases in the network. (Note that in solving the nonlinear pressure drop equations, it is possible for iterates on the minor arc rates to result in recirculating flows; if this occurs, the iterate is rejected and the Newton step is adjusted to avoid the problem.)

While the pressure loss equations (Eqs. 13 - 19) are valid for any type of flow (under the assumption that at any location, pressures in all of the phases are the same), care must be taken in their construction, since the pressure drop between nodes is pressure and path dependent. That is, prior to convergence, the pressure drop obtained by starting at the outlet and working backwards to one of the reservoir nodes, is not the same as the pressure drop obtained by starting at the reservoir node and working towards the outlet. This has a major impact on how we form the pressure equations across the minor arcs for flow of compressible or multiphase fluids; first, even if rates are specified at the reservoir nodes, pressures cannot be computed explicitly - they must be included in the set of unknowns solved by Newton Iteration; and second, the successful convergence of the network problem for the minor arc rates depends on how the pressure equations are formed. We have enjoyed success using the following procedure. To form the pressure equations for the minor arcs connected to the reservoir nodes, we start at the specified (or assumed) pressure at each reservoir node and compute pressure drops along the major arcs leading to the outlet. For the interior minor arcs (i.e., minor arcs that are not connected to reservoir nodes), we start at the outlet pressure and work backwards along the paths of major arcs connected to each end of the minor arc. Thus for example, if we wanted to calculate the pressure drop across minor arc 8 in Fig. 3, we would start from the outlet pressure (node a) and work back to nodes b, f and g, then start at node a and work backwards to nodes b, c, and g; (at convergence, the two paths would give the same pressure at g). If mass rate is specified at node a, calculations for the interior minor arcs proceed from the outlet using the last calculated estimate of the outlet pressure from the reservoir node calculations.

If a phase rate is specified at the outlet, the problem becomes slightly more complicated. As in the case of single phase flow, we convert one reservoir arc from a minor to a major arc and add a mass rate constraint equation. In the following, we describe the additional mass constraint for the cases of specified oil, gas and water rates at the outlet node.

Specified Oil Rate

If the oil rate at the outlet is specified as $q_{o,\text{specified}}$ STB/d, the oil mass rate at the outlet is given by

$$\dot{m}_{o,\text{outlet}} = q_{o,\text{specified}} \rho_{o,sc} \tag{20}$$

where $\rho_{o,sc}$ is the density of the stock tank oil in lbm/STB. At inlet reservoir node i, the total mass rate $\dot{m}_i$ is related to the oil rate at that node via

$$\dot{m}_i = \dot{m}_{o,i}\left(1 + \frac{GLR_i\left(1+WOR_i\right)\rho_{g,sc}}{\rho_{o,sc}} + \frac{WOR_i}{\rho_{o,sc}}\rho_{w,sc}\right) \tag{21}$$

so

$$\dot{m}_{o,i} = \frac{\dot{m}_i \rho_{o,sc}}{\left(\rho_{o,sc} + GLR_i\left(1+WOR_i\right)\rho_{g,sc} + WOR_i\rho_{w,sc}\right)} \tag{22}$$

and our mass constraint equation becomes

$$\sum_{i=1}^{N_{RES}} \frac{\dot{m}_i}{\left(\rho_{o,sc} + GLR_i\left(1+WOR_i\right)\rho_{g,sc} + WOR_i\rho_{w,sc}\right)} = q_{o,\text{specified}} \tag{23}$$

where N_{RES} is the number of reservoir nodes that admit flow to the network. For our example network, we would write

$$\frac{\dot{m}_{15}}{\left(\rho_{o,sc} + GLR_n\left(1+WOR_n\right)\rho_{g,sc} + WOR_n\rho_{w,sc}\right)} = q_{o,\text{specified}} - \frac{\dot{m}_{16}}{\left(\rho_{o,sc} + GLR_o\left(1+WOR_o\right)\rho_{g,sc} + WOR_o\rho_{w,sc}\right)} - \frac{\dot{m}_{17}}{\left(\rho_{o,sc} + GLR_p\left(1+WOR_p\right)\rho_{g,sc} + WOR_p\rho_{w,sc}\right)} - \frac{\dot{m}_{18}}{\left(\rho_{o,sc} + GLR_q\left(1+WOR_q\right)\rho_{g,sc} + WOR_q\rho_{w,sc}\right)}$$

Specified Gas Rate

For specified gas rate $q_{g,\text{specified}}$ scf/d, the gas mass rate at the outlet is given by

$$\dot{m}_{g,\text{outlet}} = q_{g,\text{specified}} \rho_{g,sc} \tag{24}$$

where $\rho_{g,sc}$ is the density of the produced gas in lbm/scf. At inlet reservoir node i, the total mass rate $\dot{m}_i$ is related to the gas rate at that node via

$$\dot{m}_{g,i} = \frac{\dot{m}_i GLR_i\left(1+WOR_i\right)\rho_{g,sc}}{\rho_{o,sc} + GLR_i\left(1+WOR_i\right)\rho_{g,sc} + WOR_i\rho_{w,sc}}\,. \tag{25}$$

In this case, our constraining equation becomes

$$\sum_{i=1}^{N_{RES}} \frac{\dot{m}_i GLR_i\left(1+WOR_i\right)}{\rho_{o,sc} + GLR_i\left(1+WOR_i\right)\rho_{g,sc} + WOR_i\rho_{w,sc}}\,. = q_{g,\text{specified}} \tag{26}$$

Specified Water Rate

For specified water rate $q_{w,\text{specified}}$ scf/d, the water mass rate at the outlet is given by

$$\dot{m}_{w,\text{outlet}} = q_{w,\text{specified}} \rho_{w,sc}\,. \tag{27}$$

where $\rho_{w,sc}$ is the density of the produced water in lbm/STB.

At inlet reservoir node i , the total mass rate $\dot{m}_i$ is related to the water mass rate at that node via

$$\dot{m}_{w,i} = \frac{\dot{m}_i WOR_i \rho_{w,sc}}{\rho_{o,sc} + GLR_i \left(1+WOR_i\right)\rho_{g,sc} + WOR_i \rho_{w,sc}}. \quad (28)$$

and our constraint equation assumes the form

$$\sum_{i=1}^{N_{RES}} \frac{\dot{m}_i WOR_i}{\rho_{o,sc} + GLR_i \left(1+WOR_i\right)\rho_{g,sc} + WOR_i \rho_{w,sc}} = q_{w,\text{specified}}. \quad (29)$$

Pressure Equations

In the case of a specified outlet rate, care must be taken in forming the pressure loss equations, since the outlet pressure does not explicitly appear. Since multiphase PVT properties have a large impact on pressure losses in the network, guessed values of the outlet pressure must have an impact on the final solution if any iteration scheme for solving the network equations is to be successful. We incorporate the outlet pressure in forming the pressure loss equations as follows. We start at each reservoir node where pressure is specified and compute pressure losses along the major arcs to the network outlet, then we work backward along the major arcs to the neighboring inlet reservoir node and form our pressure difference equation as the difference between the actual neighbor node pressure and the computed value. We repeat this sweep for all reservoir nodes where pressure is specified (except the last, since it has no neighbor to sweep backwards to). Then, we sweep forward from the last pressure specified reservoir node to the outlet, and backward (along major arcs) to the internal nodes connecting each non-reservoir internal arc. The final pressure difference equations are formed by differencing the computed pressure loss across the internal minor arcs and the pressure losses obtained across the arc from the backward sweeps from the last computed outlet pressure. In this manner, convergence on the final network mass rates also guarantees convergence on the outlet pressure.

APPLICATION

Significant benefits are being reported by operators around the world from the use of inflow control devices (ICDs) in well completions. ICDs have been demonstrated to generate additional recovery in long horizontal well applications. ICD technology can offer double digit oil recovery improvements and the best results are obtained when the installation is optimized properly. The capability of properly simulating the wellbore and reservoir interaction is considered as being highly beneficial in the design and analysis of advanced well completions. As there is great complexity of the flow topology throughout the completion, a proper evaluation of these completions requires a flexible approach like the network approach presented in this paper. The technology has the unique ability to handle the most advanced completions and well paths including multiple laterals, multiple annuli and any combination of screens and ICD's.

When it is coupled with a reservoir flow simulator, the presented network simulation approach allows detailed study of the completion's impact on time dependent phenomena like water and gas coning and can be used to forecast production for different completion alternatives. However, used as a stand-alone program with simple analytical expressions to emulate a steady state reservoir response, the completion network model can still be used to illustrate the effect on steady state inflow profiles for different completion strategies. Example prediction of a horizontal well with and without static inflow control devices is given below.

A 1000 ft. long horizontal well was simulated with and without ICDs in the base pipe. The wellbore diameter is 0.3 ft and the internal diameter of the base pipe is 0.2 ft; i.e., there is an open annulus between the basepipe and the reservoir formation. Connecting the basepipe to the annulus, we have either screen (with practically no resistance to flow), or ICDs. The ICD's are spaced at 40 ft intervals, so there are 25 ICDs in the well.

Figure 4 shows the pressure along the wellbore with and without ICDs. For the case without ICDs, the reservoir experiences pressures identical to those in the basepipe, while in the ICD case, reservoir pressures are isolated from the basepipe pressures due to the pressure loss through the ICDs. For the case with no ICDs, it can be seen that the sand-face pressure is very uneven due to the pressure losses along the wellbore. Thus, more fluid will flow into the heel of the well, with less entering at the toe. For the ICD case, the sand-face pressure profile is much more even, which causes uniform fluid flow from the reservoir to the well.

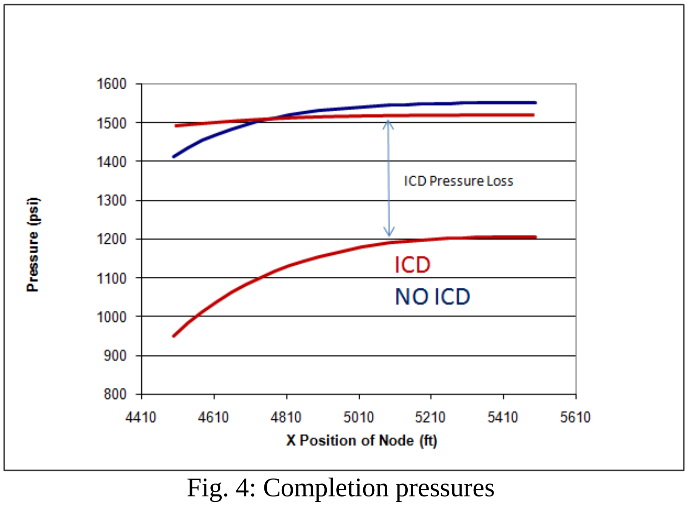

Fig. 4: Completion pressures

Figure 5 shows the inflow profiles along the wellbore for the same cases. It is seen that the uneven drawdown for the case with no ICDs causes a very uneven inflow profile where most of the inflow comes from the heel part of the wellbore. Uneven inflow may cause premature breakthrough of water and gas. The ICD case has a much more even inflow profile as could be expected from a more even sand-face pressure profile.

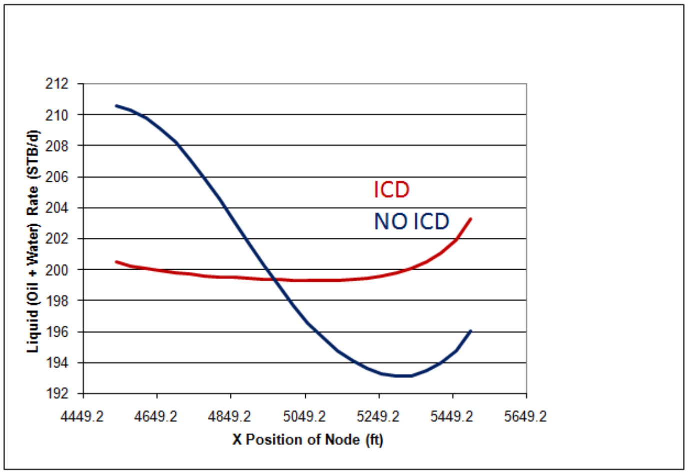

Fig. 5: Sand-face inflow profile

Significant axial flow in the well annulus is often associated with large wellbore pressure losses; this is because axial annulus flow tends to pack the annulus with solid fines from the reservoir and cause plugging of screens. Isolating the annulus from the base pipe using ICDs significantly reduces axial annulus flow. This is illustrated in Figure 6. The ability to quantify and illustrate these phenomena is a major benefit of using the network simulation to model flow in near-wellbore completions.

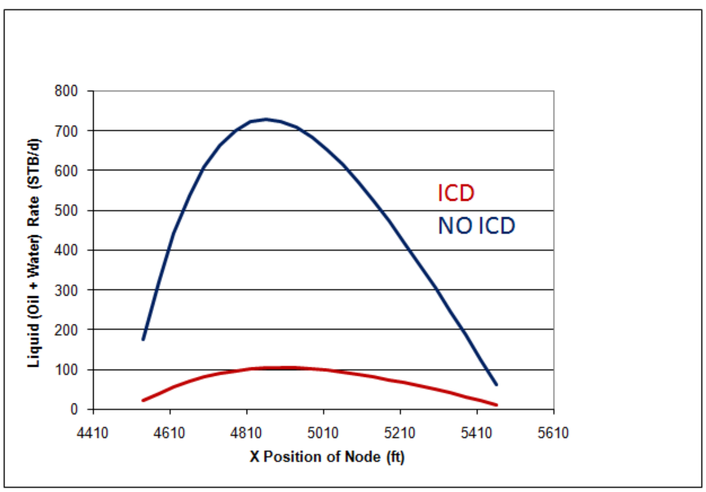

Fig. 6: Axial Flow along the annulus.

CONCLUSIONS

In this paper we have presented an efficient method for computing multiphase flow in near-wellbore networks. The method involves decoupling the mass and pressure loss equations, and solving for mass flows in different parts of the network (i.e., “minor” and “major” arcs), in a sequential manner rather than solving for all rates simultaneously. This approach results in a very efficient network solver that can be used in a coupled reservoir flow/ well bore simulator. We illustrated the utility of the method with a horizontal well example.

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